

User Guide: Understanding FEA Stress and Fatigue Mechanics

A Seminar for Simulation Engineers

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1. WHY WE ARE HERE: STRESS IS EASY, FATIGUE IS SCARY

Most Engineering failures are caused by fatigue, fatigue failure is the fracturing of a given material due to cracks induced from cyclic stresses. What makes fatigue so dangerous is that the stress levels that causes fatigue damage are much lower than the tensile and yield strength of the material. This webinar will discuss fatigue and how you can use FEA to prevent failures by using stresses extracted from the model to calculate the expected life of a product based on the material properties. In order to extract meaningful stress values, it is vital to have a robust clean mesh that will allow for accurate contouring of stresses.

1.1 WHAT WE WILL COVER

- It is all about “stress” – Quality Stress Results
- Accuracy of the Fatigue Calculation
 - It is a global question; i.e., how accurate are your loads?
- Are you chasing stress at a stress artifact? (sharp corner, fictional loading (point loading))
- Is your stress value robust?
- Basic, first order high-cycle fatigue is simple if you can keep focused to the bigger picture
- Fatigue starts from surface defects; therefore focus on the surface stress
- Multiple stress states can be complex. We’ll talk briefly and then run for the exit

1.2 WHAT WE DON’T COVER

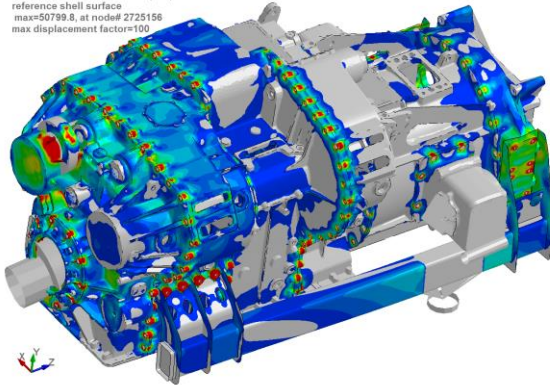
- Low-cycle fatigue damage (plastic damage)
- Multi-axle fatigue damage

2. WHAT WE KNOW ABOUT STRESS AND FATIGUE

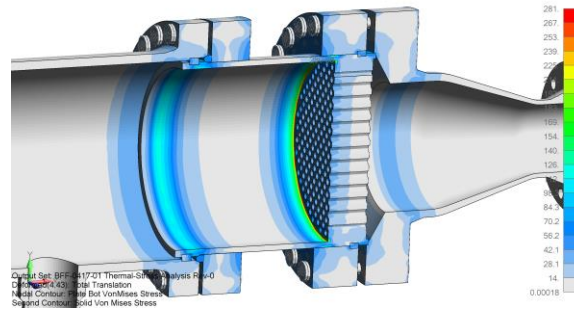
A selection of some consulting projects at Predictive Engineering where fatigue has played a role:

3,000 HP 8-Speed Transmission

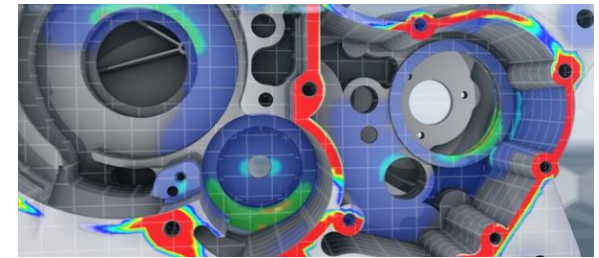
Roadmap Linear and Nonlinear Analysis - 3,000 HP Transmission
Time = 1
Contours of Effective Stress (v-m)
reference shell surface
max=50789.8, at node# 2725156
max displacement factor=100



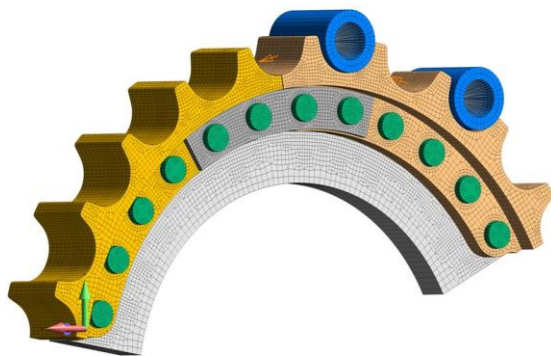
Bolted Flanges



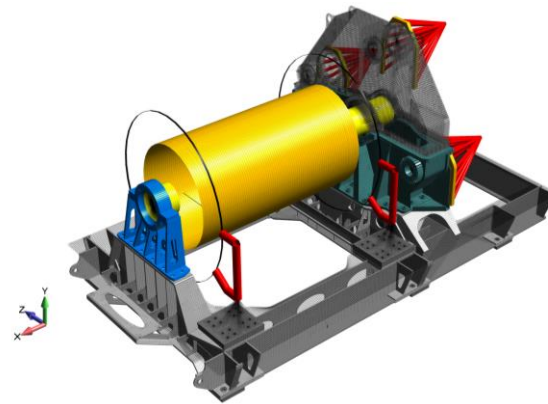
Norton Motorcycle Crankcase



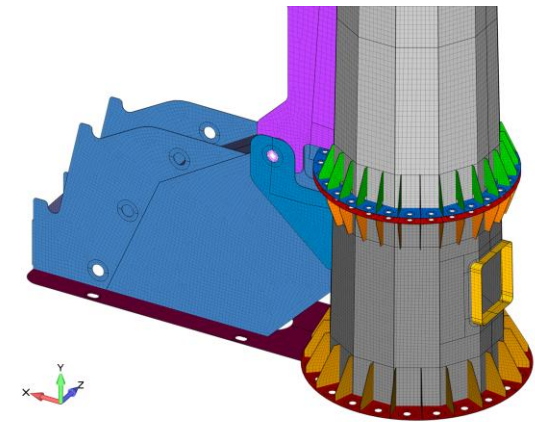
Open-Pit Mining Apron Conveyor



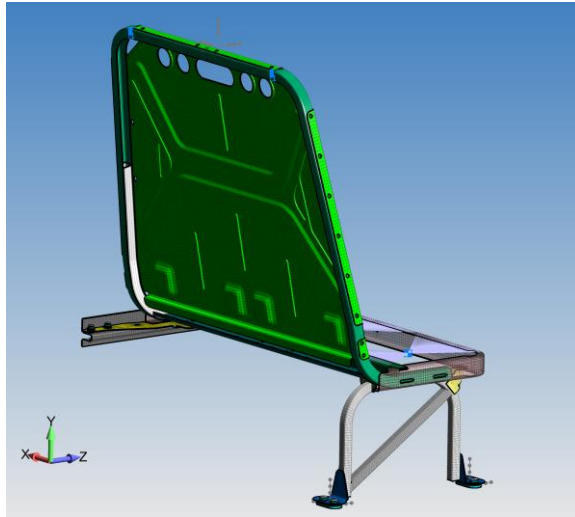
Off-Shore, High-Speed Winch



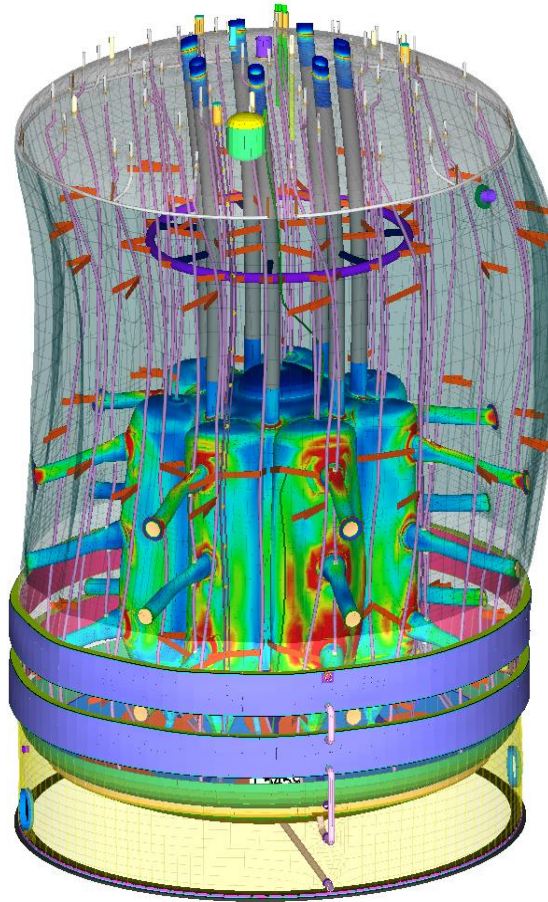
100 kW Wind Turbine Tower



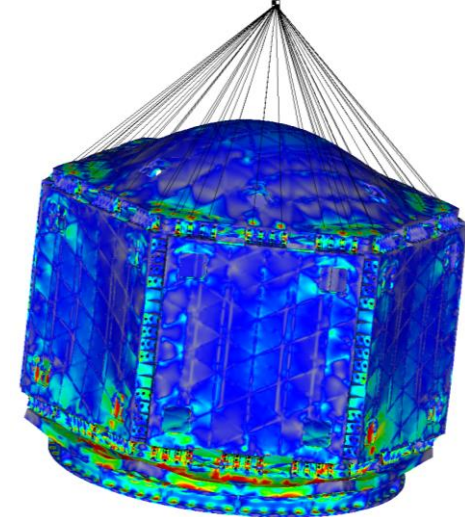
Bus Seat



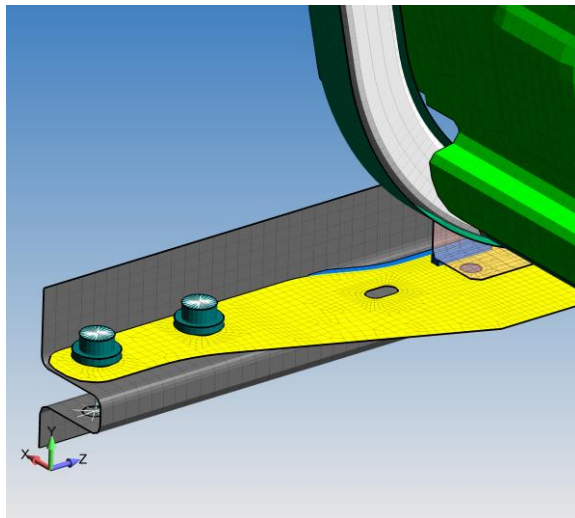
Seismic Analysis



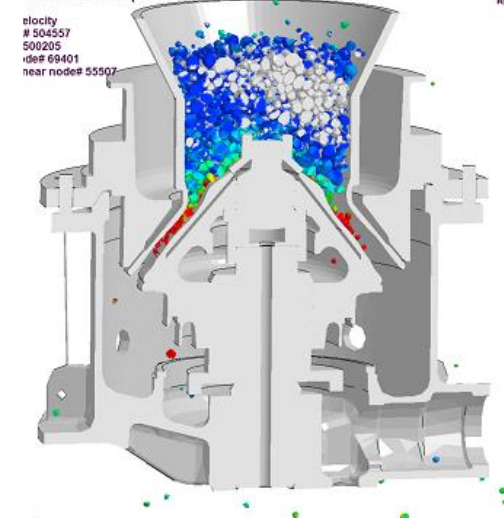
PSD Analysis of Cube Satellite



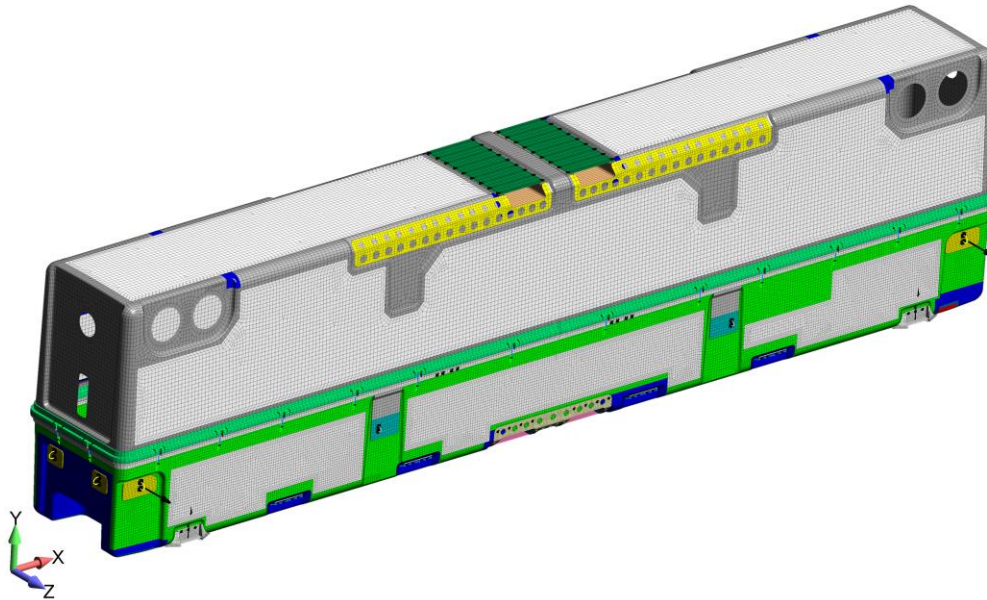
Close-Up View of Bolted Connection



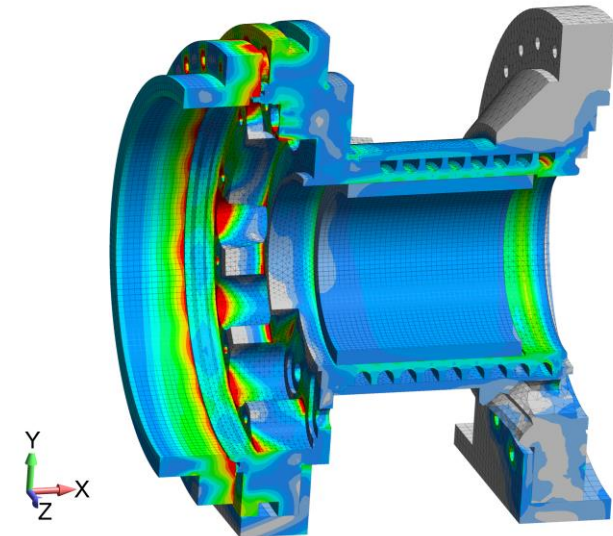
High-Speed Cone Crusher



Composite Aviation Container

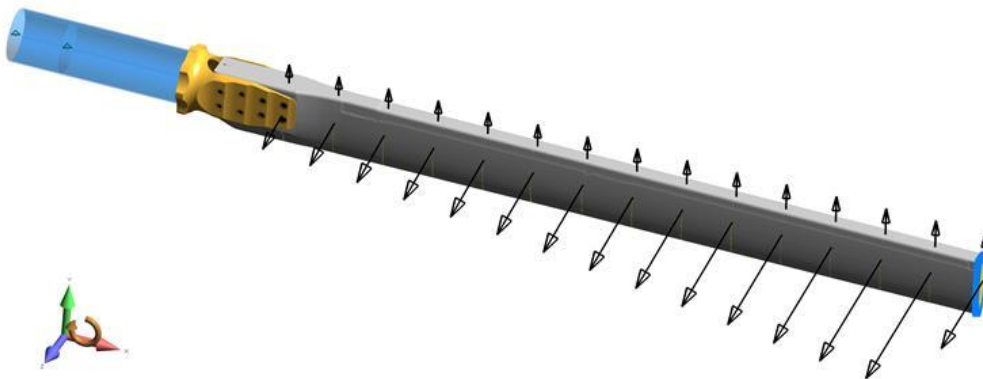


Magnetic Bearing 500 kW Generator

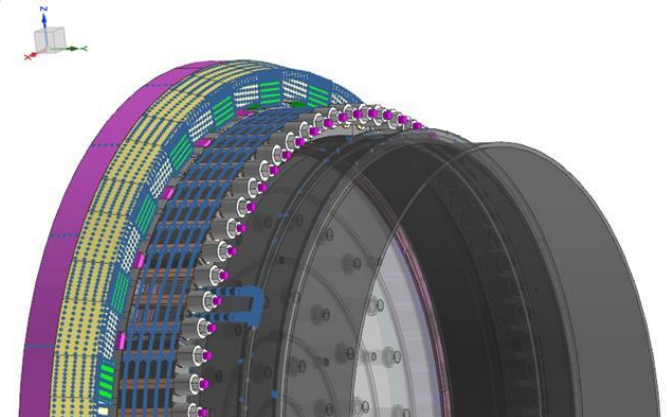


Output Set: Thermal-Stress in 500kW Turbine Generator
Nodal Contour: Nonlinear Solid Von Mises Stress

Helicopter Tail Rotor with Bolted Connection



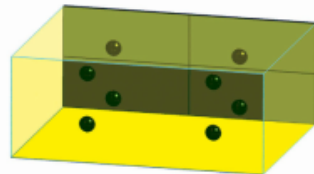
High-Temp Furnace Bolted Connection



3. STRESS MECHANICS

3.1 WITHOUT A FOUNDATION, IT IS DIFFICULT TO TACKLE MORE DIFFICULT SUBJECTS

Analysis /// FEA Visualization



See Analysis Data's True Colors

Why you don't always get what you want—and how to get what you need.

BY GEORGE LAIRD AND PAMELA J. WATERMAN

“Whenever you see a stress contour plot, just assume that it is wrong,” says Mark Sherman, head of the Femap Development Team for Siemens PLM Software Solutions. Although Sherman’s comment sounds a bit dramatic, it’s par for the course in computer modeling, where a common saying is “garbage in, gospel out (GIGO).” The questions that these comments raise are simple to pose, but are somewhat vexing to answer, even for the specialists.

How does a user quickly check finite element (FE) stress data validity or accuracy? How can one astutely defend these colorful, high-resolution results against the casual interloper who is throwing darts? Is the data gospel or garbage? This article will give you the background and ammunition to defend your results.

How Stresses are Calculated in FEA

Let’s look behind the scenes at the general approach to FE analysis. Once you have prepped and launched your FE model, the software analysis engine kicks into action, taking each element and breaking it down into a series of simple stiffness equations. Whether linear or non-linear, the equations are assembled into a massive matrix, solved by one of several intensely mathematical methods and evaluated for the given constraints and loads.

With a quick menu-click and perhaps some rotations, you have a colored contour graph showing ... *what exactly?*

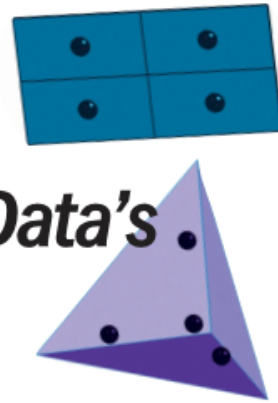


FIGURE 1: Gaussian integration points are shown for common finite element shapes: plate, brick and tetrahedral. Images courtesy of Predictive Engineering

Time to back up. What is glossed over in most explanations is how the analysis engine generates stiffness equations from oddly shaped elements defined in two or three dimensions. It is an amazingly beautiful process of simplification done over several stages.

First, each element is broken down into quadrants. A weighting formula is used to calculate the approximate volume of each element using simple polynomials. Known as Gaussian integration, this step is the cornerstone of all FEA technology. Without this simplification, FEA would not exist.

Second, the software generates stiffness equations and the analysis engine applies the given constraints and loads. Then, the engine calculates all of the displacements at the element’s corner points or nodes, leading to the next question: How does it generate stresses from displacements away from the element’s corners?

The answer lies in another great trick of the FEA process. Theoreticians have determined that the best place to calculate stresses in finite elements lies at the Gaussian integration points (the center of the quadrants). The software then takes these displacements and uses Gaussian integration to calculate first the strain and then stress within each “Gaussian volume.” Most finite elements are analyzed using four Gaussian integration points, and thus the analysis engine generates stresses at four discrete

points within the element. (See Figure 1.)

But how do these Gaussian integration points relate to what we see on our real-world contour plots? Because most users have no use for raw element data, one final processing step is done. Values for the Gaussian point stresses are interpolated into the element’s center, and also extrapolated out to the corner points or nodal points. At this stage, the analysis engine has done its work—and the visualization process starts.

De-bugging Jagged Stress Contours

You are more knowledgeable now than many stress analysts about how element stresses are computed, but what exactly are you seeing? These millions of extrapolated points have been loaded into the software’s graphic display system and presented as a dazzling, yet smoothed spectrum of colors. Smoothed is a key word here.

In the default mode, FEA programs average the corner point stresses from each element and only present the averaged value to the user. This little smoothing technique has its good and bad points. Overall, it is a good thing because it smoothes out the stresses into a cleaner pattern. Numerically, this process adjusts for the non-physical variations in stresses that stem from minor variations in the element’s shape.

Because physical stress is not a step-function (it wants to flow smoothly), numerically smoothed results display the more accurate solution. If they don’t, you should definitely worry! This is the garbage part. Alarm bells should ring, which brings us to the task of developing your FEA eyes.

When contours look jagged, with lots of red spots (see Figure 2a) or have extremely irregular shapes, three causes are likely:

- poorly applied loads and constraints (likely);
- complicated or bad CAD geometry (tricky); or
- poor mesh quality. (See “Judging Good and Bad FE Shapes” to the right.)

Because we are using finite elements to approximate a continuum, sometimes it’s best just to accept a few discrepancies when they can be easily explained as a reality of the modeling process. For example, one could quote Saint-Venant’s principle and say that stress and displacement contours away from load application points, and do not depend on how the load was applied (concentrated or distributed) because forces and moments are always conserved. In other words, if the region of interest is far away from the load application point, you can ignore the less-than-smooth stresses around the load and constraint points.

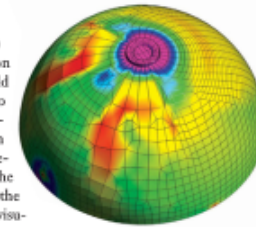


FIGURE 2A: Stress is never jumpy. It should always vary smoothly, so if your stress results don’t appear right, then most likely they are not. Here, warning lights should come on when you see the “spots” of high stress repeatedly popping up along the curved edge of this part.

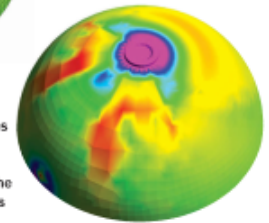


FIGURE 2B: These stress results are 20% higher than those in Figure 2a. The values reflect smooth transitions across the geometry. Visual inspection will tell you that the higher stresses reflect results much closer to reality.

From a geometric point of view, designers hand over CAD files that include every manufacturing detail down to 0.005-in. sharp edge-breaks or small diameter oil-feed passageways. Typically, we can (and should) ignore these small regions by invoking another useful relationship from Jean Claude Barré Saint-Venant. (See “Saint-Venant’s Principle of Decreasing Load Effects,” page 18.) He discovered that small features only create localized disturbances in the stress field. The extent of this disturbance is no more than three times the characteristic dimension of this small feature. For example, for holes of radius R , this size is $3 \times R$. Thus, if your objective is to determine the overall stress of the structure, localized excursions in the stress field will not affect your final answer. This can be

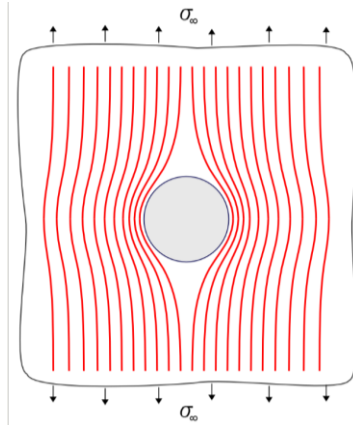
Judging Good and Bad FE Shapes

Gaussian integration is an exact method only for elements that have near-perfect shapes—side-to-side ratios of 1×1 for plates or $1 \times 1 \times 1$ for solids (bricks and tetrahedra, for example). Once the element shape starts to degrade, so does the quality of your results.

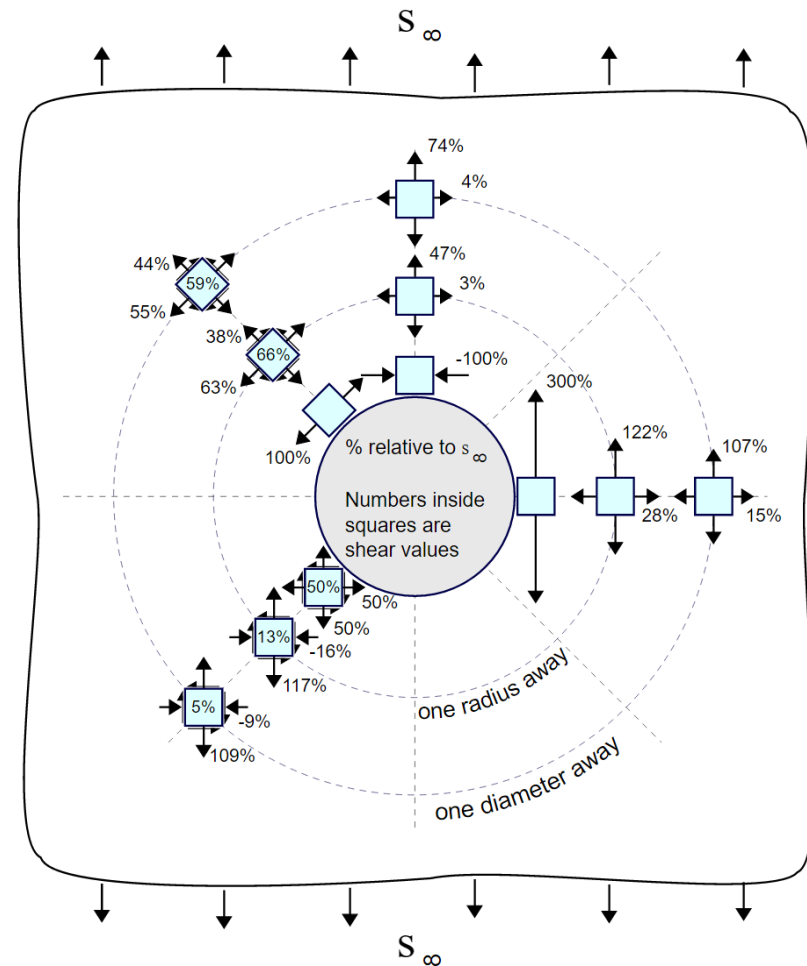
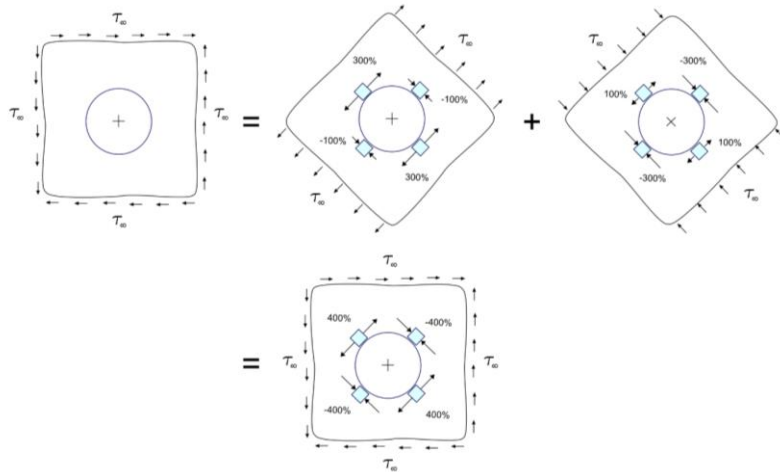
A standard rule of thumb is that the element shape should be “pleasing to the eye” and maintain this regularity across the model. Although it’s a subjective approach, if the mesh appears to be close to having $1 \times 1 \times 1$ proportions, you are good to go.

3.2 LET'S START WITH THE OBVIOUS – THERE IS A HOLE – THE BASIS OF ALL FATIGUE CALCULATIONS

Stress Flows
$$\frac{\partial^4 \phi}{\partial x^4} + 2 \frac{\partial^4 \phi}{\partial x^2 \partial y^2} + \frac{\partial^4 \phi}{\partial y^4} = 0$$



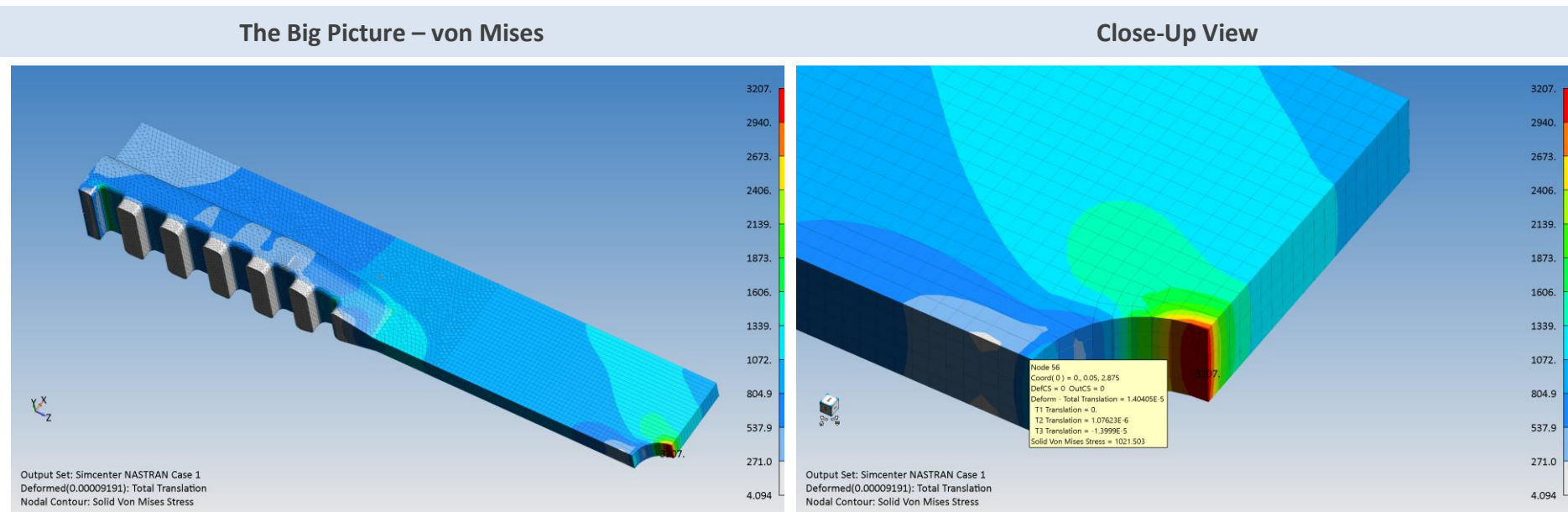
With Pure Shear – Stress Concentration 4x



Images courtesy of www.FractureMechanics.org

3.3 LET'S PLAY WITH AN EXAMPLE MODEL

What does stress mean – it is force over area. Is von Mises telling you the whole story? What does a negative von Mises indicate?



3.4 SOME STRESS MECHANICS TO THINK ABOUT?

- Under tension we have a von Mises stress $\sim 1,000$ psi at the top of the hole;
- Is it tensile or compressive? What happens if we put the sample under compression? (Model / Output / Calculate)
- Is the stress converged? How does the centroid and nodal stresses differ?

4. FATIGUE – 1ST ORDER

4.1 LET'S JUST CUT TO THE CHASE IS YOUR MAXIMUM STRESS < 80% YIELD STRENGTH?

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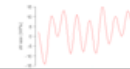
Engineering Mechanics White Paper



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1. AN OVERVIEW

The FEA marketplace offers many complex and extremely detailed fatigue programs. Given this reality, why would the world need another fatigue program? From an engineering perspective, the requirement to prevent fatigue failure in high-cycle environments constitutes about 90% of the demand. The remaining 10% is within the realm of short-cycle, plastic strain events, and the interpretation of fatigue damage for such events is quite difficult. From this basis of serving the 90%, we present an essential fatigue program that performs ASTM-type rainflow counting and allows the user to directly enter or chose from a library, common fatigue data relationships. The program's workflow leverages the Femap interface and with its directness and ease-of-use, encourages the practicing engineer to more actively consider fatigue in their analysis work.

2. INTRODUCTION

A brief walk-through is given on how a fatigue analysis works and a bit of foundation knowledge to guide a new user through this process. It should be mentioned that we are focusing on the high-cycle fatigue of metals. Just to ensure that we are all on the same page, the difference between low-cycle and high-cycle fatigue is briefly summarized in Table 1.

Table 1: A quick summary of the difference between Low-Cycle and High-Cycle Fatigue

| Low-Cycle Fatigue (Strain-Life) | High-Cycle Fatigue (Stress-Life) |
|---------------------------------|----------------------------------|
| Stress > 80% σ_{Yield} | Stress < 80% σ_{Yield} |
| Cycles < 10,000 | Cycles > 10,000 |

Since most design work focuses on structures with near infinite life, the stress target is typically 80% of the material's yield strength (σ_{Yield}) or lower. This requirement makes the stress-life approach a natural fit.

As a side note, one should not consider this article as the "last word" or even a "complete word" about the fatigue process. There are dozens of handbooks on fatigue analysis, and if one would like to become proficient in this branch of engineering, it can take years of study and perhaps a master's or a Ph.D. of engineering along the way. Heretofore, our objective in this note is just to provide a common foundation of understanding from which to launch more complete discussions.

2.1 THE PROCESS

For clarity, the fatigue process is broken down into five sequential steps:

- Stress calculations, whether by hand or turning the FEA crank;
- Sketching out the load events to create load cycles;
- Form logical pairings of maximum and minimum stresses between load sets (Rainflow);
- Calculate damage for each load pairing from fatigue curve;
- Sum damage using Miner's rule.

4.1.1 WHY FATIGUE IS SNEAKY AND FEARED



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2.2 WHAT IS FATIGUE IN METALS?

This is not meant to be a treatise but just enough to whet your interest in the theme of fatigue theory, and its application.

Fatigue starts with the movement of dislocations within the metal's crystal lattice. These dislocations pile up along grain boundaries, impurities (i.e., oxides), secondary hard phases (e.g., the silicon network within A356 cast aluminum alloys) and interstitial compounds or just in general, anything that is not part of the pure crystalline metallic matrix. Over thousands and thousands of cycles, these dislocations pile up to such an extent that a network of microscopic cracks is created within the material. Once this network of cracks has formed, the fatigue process speeds up significantly with these small cracks bridging together into larger cracks and finally zipping along to form a final large massive crack where the structure unexpectedly fails. The failure is termed unexpected since nobody thought that the stresses were excessive since they had designed to 50% of the yield/ultimate strength of the material or some other "rule-of-thumb".

If one is of the curious sort, it begs to question how the 50% rule-of-thumb got started. In many handbooks, Figure 1 aptly describes the relationship between alternating stress and the number of cycles to failure for ferrous and non-ferrous materials. Ferrous materials (steel) exhibit a plateau while non-ferrous materials (aluminum, brass, magnesium, etc.), will eventually fail given billions and billions of cycles. Of course, exceptions occur and in practice, here is a short list:

- Occasional overloads or impacts can destroy the ability of the material to have a fatigue limit;
- Corrosion (a common rational offered-up sometimes by metallurgists to explain unexpected failures);
- High-temperatures that can introduce microstructural changes.

In the special case of non-ferrous materials, it is more common to specify fatigue strength (S_f) as stress per number of cycles to failure. As example, a manufacturer of aluminum A356-T6 truck hubs uses a design limit of $S_f = 100$ MPa with an estimated $1e10^8$ cycles to failure. For a standard commercial long-haul truck, it's enough to satisfy their clients' fatigue requirements.

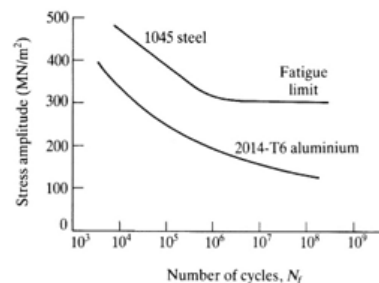
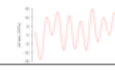


Figure 1: A standard representation of an S-N (stress-cycles) curve for typical ferrous and non-ferrous materials. The fatigue limit for ferrous materials is roughly $\frac{1}{2}$ the material's ultimate strength.



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Another way to think about this 50% rule is to look at the mechanics of a void within a large body. Standard mechanics calculate the stress concentration (K_t) of a spherical void within a large body as 2.0. The FEA model in Figure 2 provides a visualization of these mechanics in color.

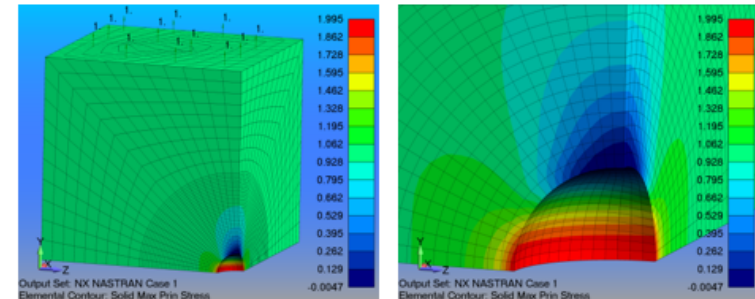


Figure 2: A symmetric block is given a uniform pressure load of -1.0. The FEA model shows a maximum stress of 1.995.

Since all materials contain small defects, it is easy to imagine that when designing to 50% of the yield strength, the true stress at microstructural defects is at 100% of the material's yield strength. Not to belabor this point but since this is a material's discussion and the yield strength of a ferrous/non-ferrous material is based on the empirical observation that when the load is released, no observable plastic deformation is noted, but in reality, extensive dislocation movement occurs at stresses greater than 50% of the yield strength of the material (σ_y). Hence, even before the material reaches its σ_y dislocations are moving, combining, clustering and causing nano-sized cracks in the crystalline structure. Given this basis, whenever the load is greater than 50% σ_y , we have dislocations moving through-out the material and near defects, causing rather massive localized plasticity. This is the essence of material fatigue and why every test sample will fail at a different number of cycles due to metallurgical imperfections.

2.2.1 SURFACE TREATMENTS AND THE USE OF STRESS MODIFICATION FACTORS

Most fatigue data is obtained using polished samples to enforce consistency and minimize variability within the data set. The challenge for the engineer is how to use fatigue data while accounting for non-polished surface conditions and/or different surface treatments. Engineered structures are rarely polished and are sometimes subjected to harsh environments that tend to scratch the surface. Additionally, the surface may be specially treated to enhance its hardness and/or fatigue properties. The easy route is just to create your own fatigue data set that would cover the intended surface roughness or treatment. But since most projects don't have dedicated budgets for fatigue data, we are left with using stress modification factors to adjust calculated stress data and then apply these values toward existing *in-hand* or published data.

4.1.2 SURFACE EFFECTS AND UNDERSTANDING MEAN STRESS



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It should be mentioned that the fundamental reason why surface treatments are so critical in fatigue is that for most engineered structures, the highest stress occurs on the surface due to bending and torsion stresses. Likewise, surfaces get scratched or as one commentator noted, the structure is subjected to the "blunt axe" treatment. This combination of base maximum stress and surface defects makes most fatigue experts run to apply the highest possible factor that they can reasonably justify. These factors can be broken down into two categories: (i) Surface Roughness and (ii) Surface Treatments. The first category is more directed toward modifying the base stress number since these factors increase the calculated stress. The second category is directed toward the established fatigue curve since one is modifying a characteristic of the base material, i.e., surface treatment. This can be as easy as using a stress modification factor or as complex as shifting the base fatigue curve via some algorithm that accounts for the material variability of the surface treatment. However, these distinctions are a bit academic since at the end of the process, one is multiplying the base stress number by a factor.

2.2.1.1 SURFACE ROUGHNESS

Manufacturing processes such as casting, forging, machining, etc. create unique surface profiles. The roughness of the surface is generally a compromise between cost and engineering requirements. As the surface finish improves the fatigue life increases albeit manufacturing costs go up. Figure 3 shows a plot of surface factor (i.e., fatigue life) against tensile strength. As the surface finish improves (smaller numbers are better), so does fatigue life as indicated by higher surface factor numbers. The utility of Figure 3 is that it demonstrates that as the steel's strength increases, the surface finish is more and more important.

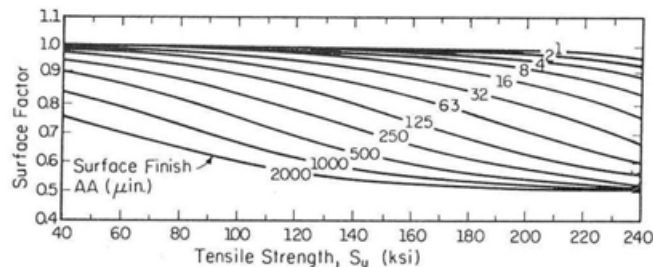


Figure 3: Surface factor (fatigue life) given a certain surface finish.

2.2.1.2 SURFACE COATINGS AND TREATMENTS

Surface coatings such as chrome plating, anodizing, cadmium plating, etc. can have detrimental effects with respect to fatigue. Testing is required to determine the specific fatigue curve de-rating (knockdown) for a given coating/base material combination. In general, the typical numbers are between 10-30%.

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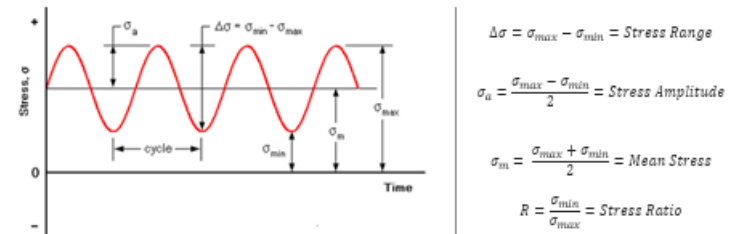


Figure 4: This sketch lays down the foundation of how stresses within a cyclic event are described within the world of fatigue terminology.

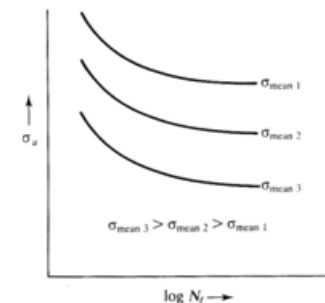


Figure 5: As the mean stress (σ_m) increases, the alternating stress (σ_a) to failure decreases.

In the majority of S-N curves presented in the literature, a common denominator is the use of stress amplitude (σ_a) to indicate the driving stress to failure with no mention of the mean stress (σ_m). This can cause a few problems for someone new to the field in trying to decipher the utility of the presented data since σ_a by itself doesn't paint a very complete picture. The reality is that if no other information is presented, then an S-N curve showing σ_a versus cycles (S-N) is always at a stress ratio of $R = -1.0$. The reason that some S-N curves typically only provide σ_a with no mention of stress ratio is that generating fatigue data is very expensive and requires a large data set for good statistical accuracy. One of easiest methods to generate fatigue data is the ASTM rotating beam test where a cylindrical test specimen is polished and mounted as shown on the left-hand side of Figure 6. This type of test is easy to operate, and since the stress ratio is fixed at $R = -1.0$, only one data set is generated. Hence, when no other information is presented, it is highly likely that the given data is at $R = -1.0$ and that the mean stress (σ_m) is zero. If the effect of σ_m is required, one needs to use a more complex

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4.1.3 CORRECTING FOR MEAN STRESS



setup, as shown on the right-hand side in Figure 6. As one can imagine, the resulting data set is much larger and more cumbersome to process.

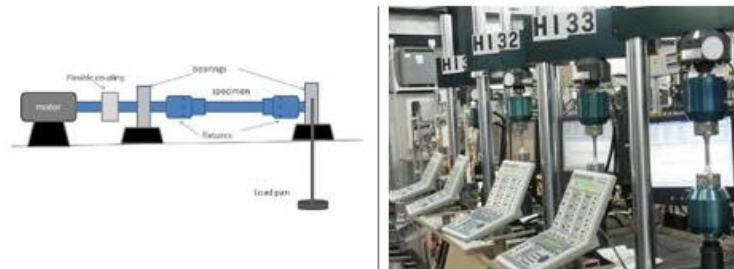


Figure 6: Classic ASTM rotating beam fatigue test providing a $R = -1.0$ and the more modern suite of fatigue test machines that can cycle the sample at nearly any stress ratio.

2.4 CORRECTING FOR MEAN STRESS

Traditionally, the industry has lacked arrays of instrumented testing machines as shown in Figure 6 and had to rely on the basic rotating beam test where the data was always at $R = -1.0$ and $\sigma_m = 0.0$. This presented a rather serious problem since it was well known that σ_m would significantly lower the fatigue life of the structure. To leverage the large and economical database of $R = -1$ fatigue data, several scientists over the years have developed empirical relationships that allow the correction of fatigue data at other σ_m values. The most popular of these corrections is the Modified-Goodman developed in the early 1900's (see Dowling's paper in Section 6, Suggested Readings). More recent work in the 1970's by Walker and Smith-Watson-Topper (SWT) provide formulas that are considered more accurate (see Dowling's paper). Figure 7 shows an application of the Goodman and SWT σ_m correction based on an original data set of $R = -1.0$. The process is to start with a base equation relating alternating stress $\sigma_a^{R=-1.0}$ to cycles to failure (N_f). The equation format is generic and provides a nice fit to most metallic fatigue data up to the point of the material's endurance limit. The mean stress corrections (σ_a^*) are then inserted as the corresponding value of $\sigma_a^{R=-1.0}$. As shown in Figure 7, as the mean stress increases from $\sigma_m = 0.0$ ($R = -1.0$) to $\sigma_m = \sigma_u/2$ ($R = 0.0$), the fatigue curve shifts downward as previously shown in Figure 5.



$$\sigma_a^{R=-1.0} = \sigma_f (2N_f)^b$$

$$\sigma_f = 1758 \quad b = -0.098$$

$$\text{Goodman } \sigma_a^* = \left(\frac{\sigma_a \sigma_u}{\sigma_u - \sigma_m} \right)$$

$$\text{SWT } \sigma_a^* = ((\sigma_m + \sigma_a) \sigma_a)^{0.5}$$

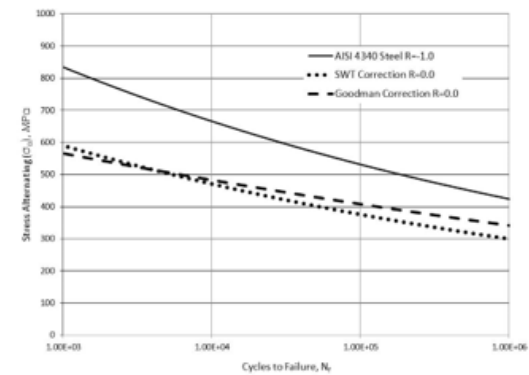


Figure 7: Starting with experimental data at $R = -1$, a mean stress correction to $R = 0.0$ is done using the SWT and Goodman equations.

4.1.4 UNDERSTANDING FATIGUE CURVES – IT TAKES TIME AND A STATISTICAL MINDSET



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A more common way to present fatigue data obtained at different σ_m levels is by plotting the maximum stress (σ_{max}) against N_f . Figure 8 presents data from the MMPDS based on this format. When data is available at different σ_m levels (or stress ratios), one can obtain a better correction using the Walker equation by fitting the exponent to the data set. For example, the SWT equation uses a fixed exponent of 0.5 while the Walker equation presented in Figure 8 is derived from the data set and is 0.63. The fit to the data is obviously better using the Walker equation but for legacy reasons the Goodman correction is still prevalent.

$$\begin{aligned} \log N_f &= 20.68 - 9.84 \log(\sigma_{max}^*) \\ \sigma_u &= 45 \text{ ksi} \end{aligned} \quad \begin{aligned} \text{Goodman } \sigma_{max}^* &= \left(\frac{\sigma_{max} \sigma_u (1-R)}{2\sigma_u - \sigma_{max} (1+R)} \right) \\ \text{Walker } \sigma_{max}^* &= \sigma_{max} (1-R)^{0.63} \end{aligned}$$

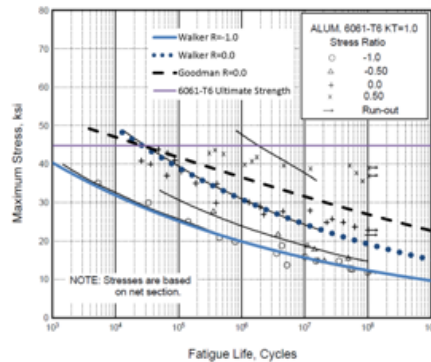
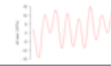


Figure 8: Working with data presented in the MMPDS, the Goodman and Walker mean stress correction is overlaid the experimental data for $R = 0.0$. One will note that the ultimate strength of the material $\sigma_u = 45$ ksi is logically never exceeded by the experimental data but that the fitted curves will incorrectly bump above this limit.

Let's now solve a more fundamental fatigue analysis problem where the designer only has the most basic of mechanical steel property data, e.g., the ultimate strength (σ_u) of the steel and needs "quick and mostly accurate" assessment of fatigue life at a stress ratio $R = 0.0$. For a broad range of steels, it is reasonable to assume that at $R = -1$, one can say that the fatigue life N_f at 1,000 cycles is $0.9\sigma_u$ and that at $N_f = 1e6$ it is $0.5\sigma_u$ (see Bannantine, Section 6, Suggested Readings and note that this is only for polished samples and real structures rarely get this lucky). This curve is given in Figure 9 along with the

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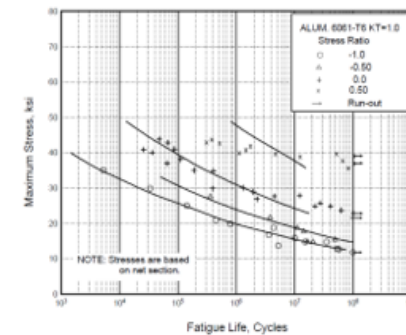


Figure 11: MMPDS data for 6061-T6 aluminum.

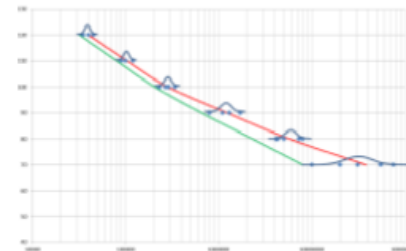


Figure 12: An example of perfectly obtained experimental data where the working curve (the line on the far left) can be created from a statistical fit of the raw data. The central line is termed the 50/50 line.

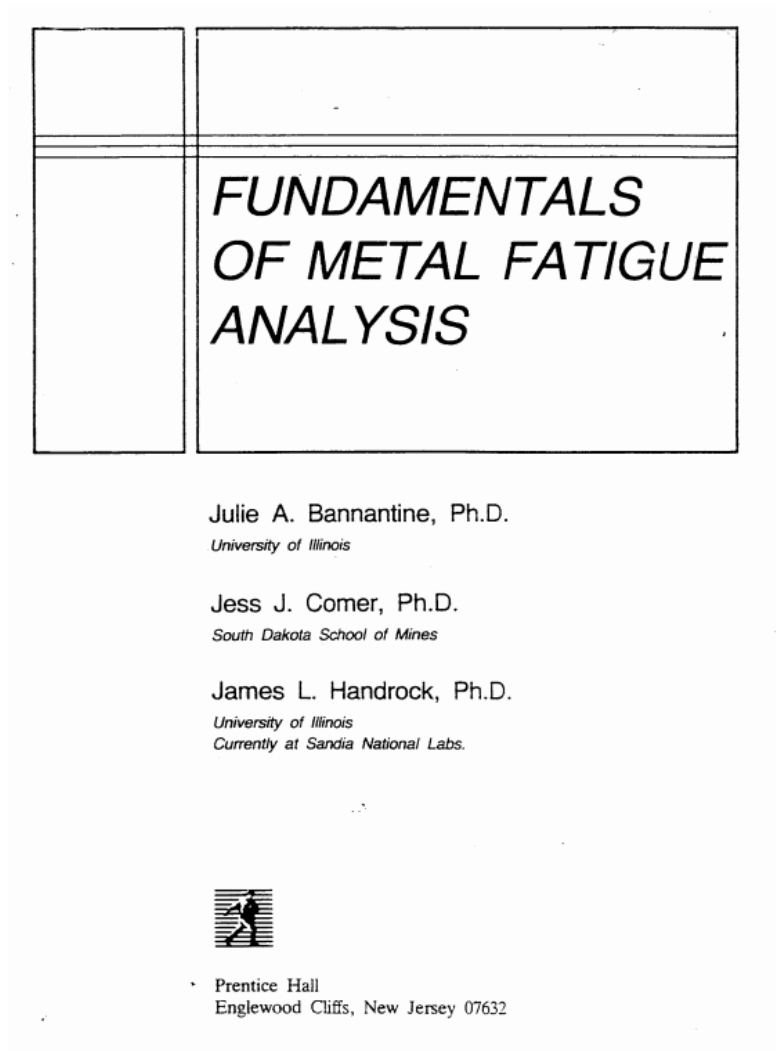
In statistical terms, the curves presented in the MMPDS and in S-N fitted data are termed 50/50 curves where one has a 50% chance that the N_f calculation will be within one standard deviation of error. The obvious challenge to this approach is that most fatigue data sets are limited and that statistical information is often lacking. Given this challenge, we have three general approaches:

- Increase the stress value (i.e., stress modification factor) used to calculate fatigue damage;
- Divide the number of calculated cycles to failure N_f by some scatter factor;
- Or a combination of the above two methods.

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4.1.5 My FAVORITE FATIGUE REFERENCE



5. CLOSURE / SUMMARY / LET'S WRAP THIS UP!

- Your fatigue calculation is only as accurate as your stress calculation;
- Little variations in stress can cause large variations in fatigue life calculations;
- All materials have defects and where there is a defect there is a stress concentration;
- Think statistically and assume variability in load and material data; i.e., adjust conservatively your numbers on the first pass. If it doesn't work, shave your margins with eyes wide open;
- The standard concept of for infinite fatigue life as $\frac{1}{2}$ the yield stress is well grounded in metallurgical reality and engineering mechanics;
- You made it. Congratulations – you now know more than 90% of your colleagues about FEA stress interpretation and the fundamentals of fatigue.

Thank you!